Thermodynamic Lower Bounds on Task-Relevant (Semantic) Information Production in Open Systems

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Abstract

Information thermodynamics has established a fundamental physical connection between entropy, free energy, and information processing. Since Landauer's principle, it is understood that information manipulation carries an energetic cost. However, standard information-theoretic measures such as Shannon entropy and mutual information quantify statistical dependence without distinguishing whether such dependence contributes to a system's objective or function. In physical, biological, and artificial systems, only specific correlations are relevant for task performance, adaptation, or control.
In this work, task-relevant (semantic) information is defined operationally as a bounded utility-weighted generalization of mutual information. Specifically, it is formulated as the expectation of pointwise mutual information weighted by a normalized utility function, thereby selecting correlations aligned with a system's objective. This definition introduces no ontological assumptions about meaning; it formalizes semantic content as task-dependent statistical alignment.
Using the established relationship between Kullback–Leibler divergence and Helmholtz free energy under canonical ensemble assumptions, we derive conditional thermodynamic lower bounds linking task-relevant information production to free energy difference and entropy production rate. Let A denote task-weighted information and define the physically relevant semantic magnitude $A_+ :=| A |$. For open non-equilibrium systems with bounded utility and Markovian dynamics, and under a positive alignment regime in which semantic growth contributes positively to total KL deviation, we obtain

$$\Delta E \geq kT\ln(2)\,\Delta A_+, \sigma \geq \lambda\,\dot{A}_+,$$

where $\sigma$is the entropy production rate and $\lambda$is a system-dependent alignment coefficient of the order of Boltzmann's constant per unit information. These inequalities are not universal thermodynamic identities; they hold conditionally when task-relevant correlation growth constitutes a subcomponent of deviation from canonical equilibrium.
A minimal two-state continuous-time Markov model is presented to illustrate how non-equilibrium fluxes generate task-aligned correlations and associated entropy production. In this setting, increasing task reward induces probability restructuring, free energy deviation,

and strictly positive entropy production, providing an analytic demonstration of the proposed bounds.
The framework does not modify existing thermodynamic laws. Instead, it refines information thermodynamics by systematically distinguishing total statistical dependence from task-aligned correlations. By introducing bounded utility weighting, it identifies a physically constrained subset of mutual information whose production is energetically limited in alignment regimes. The theory is mathematically explicit, dimensionally consistent, and experimentally testable in artificial learning systems, biological neural systems, and stochastic information engines.
In this sense, semantic information is treated not as a metaphysical construct but as a task-dependent, physically constrained component of statistical dependence. The proposed results provide a principled basis for investigating the energetic limits of functional information production in open systems.

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1. Introduction

Since Landauer's principle (Landauer, 1961), it has been physically accepted that information processing carries a thermodynamic cost. According to Landauer, the minimum energy required to irreversibly erase one bit of information is

$$\Delta E \geq kT\ln(2).$$

This result demonstrates that the relationship between information and energy is not merely analogical but physically necessary. Subsequent developments in reversible computation (Bennett, 1982), stochastic thermodynamics (Seifert, 2012), and information engines (Parrondo, Horowitz & Sagawa, 2015) have systematically elaborated the structural relationships between information, free energy, and entropy production.
However, the current framework of information thermodynamics does not explicitly address an important distinction: not all statistical information is equivalent. Shannon entropy and mutual information measure dependence between variables, yet they do not distinguish whether such dependence is valuable with respect to a system's goal, task, or function. Random correlations may increase mutual information without contributing to system performance, adaptive success, or functional output.
In physical, biological, and artificial systems, there exists a practical distinction between functionally relevant information and functionally irrelevant information. Is the energetic cost of environmentally relevant information—such as that contributing to an organism's survival—equivalent to that of random environmental correlations? Is the production of task-relevant information subject to a thermodynamic constraint different from that governing arbitrary correlation generation? This distinction has not been addressed within a clear and systematic framework in the existing literature.
The present work focuses on the following fundamental problem:
Does the production of task-relevant (utility-weighted) information obey a thermodynamic lower bound?
To address this question, the concept of "meaning" is redefined not as an ontological or subjective category, but as a task-weighted measure of information. The quantity A is

defined as mutual information weighted by a bounded utility (or loss) function and represents correlations aligned with the system's objective.
The central hypothesis of this work is formulated as follows:
In open non-equilibrium systems, the growth of task-relevant information is subject to a conditional thermodynamic lower bound.
In this context, we propose the following lower-bound hypothesis:

$$\Delta E \geq kT\ln(2)\,\Delta A_+,$$

where $A_+ := |A|$ denotes the physically relevant semantic magnitude.
This inequality is not presented as a universal physical law, but as a thermodynamic lower-bound proposal valid under specific dynamical assumptions (open systems, canonical ensemble, and Markovian transitions).
Throughout the paper, we demonstrate how this bound relates to free energy difference and entropy production rate.

The aim of this work is not to modify existing physical laws, but to extend information thermodynamics through a systematic distinction between total statistical dependence and task-relevant information. The proposed framework:is mathematically well-defined,

- is consistent with non-equilibrium thermodynamics,
- generates experimentally testable hypotheses,
- and applies to a wide range of systems, from artificial intelligence to biological information processing.

In this sense, the study provides a conceptual and mathematical refinement of the energy–information relationship by distinguishing total statistical dependence from task-relevant information production.

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2. Theoretical Background and Fundamental Concepts

This section defines the core concepts from information theory and thermodynamics used throughout this work. The objective is to demonstrate explicitly that the proposed framework is fully consistent with established physical literature.

2.1 Shannon Entropy

For a discrete random variable X, Shannon entropy is defined as

$$H(X) = -\sum_x p(x)\log p(x),$$

where $\log$ denotes the natural logarithm unless otherwise specified.
For continuous variables, the differential entropy is

$$H(X) = -\int p(x)\log p(x)\,dx.$$

This quantity measures the uncertainty of the system. As entropy increases, uncertainty about microstates increases. However, Shannon entropy does not distinguish whether information has functional or task-relevant value.

2.2 Kullback–Leibler (KL) Divergence

The divergence between two distributions $p(x)$and $q(x)$is defined as

$$D_{\mathrm{KL}}(p \parallel q) = \int p(x) \log \frac{p(x)}{q(x)}\, dx.$$

KL divergence is always non-negative:

$$D_{\mathrm{KL}}(p \parallel q) \geq 0,$$

and vanishes if and only if $p = q$almost everywhere.

In statistical physics, this quantity is directly related to free energy difference and forms the thermodynamic basis of the present work.

2.3 Mutual Information

The mutual information between two variables $X$and $Y$is defined as

$$I(X;Y) = D_{\mathrm{KL}}(p(x,y) \parallel p(x)p(y)),$$

or equivalently,

$$I(X;Y) = H(X) + H(Y) - H(X,Y).$$

Mutual information measures statistical dependence only. It does not distinguish whether such dependence is valuable with respect to a system's objective or function.

This distinction constitutes the central motivation of the present study.

2.4 Landauer's Principle

According to Landauer's principle, the minimum energy required to irreversibly erase one bit of information is

$$\Delta E \geq kT \ln(2).$$

More generally, for erasing ΔI bits,

$$\Delta E \geq kT \ln(2)\, \Delta I.$$

This relation establishes that irreversible information processing carries a physical cost. However, Landauer's principle does not distinguish between different informational contents or functional relevance.

2.5 Free Energy and KL Divergence

The Helmholtz free energy is defined as

$$F = \langle E \rangle - TS.$$

If the reference distribution $q(x)$ is the canonical Boltzmann distribution,

$$q(x) = \frac{e^{-\beta E(x)}}{Z}, \beta = \frac{1}{kT},$$

then the following relation holds:

$$D_{\mathrm{KL}}(p \parallel q) = \beta(F(p) - F(q)),$$

and therefore,

$$F(p) - F(q) = kT\, D_{\mathrm{KL}}(p \parallel q).$$

This result holds only when the reference distribution is canonical. It plays a critical role in grounding the thermodynamic interpretation of task-weighted information.

2.6 Non-Equilibrium Entropy Production

For open systems, the total entropy production rate is

$$\sigma = \frac{dS_{\mathrm{total}}}{dt},$$

and by the second law,

$$\sigma \geq 0.$$

For Markov processes, entropy production can be expressed in terms of transition fluxes (Seifert, 2012).

Local entropy reduction is possible, but it must be compensated by entropy increase in the environment. In this sense, information production is typically associated with local order formation and environmental entropy production.

2.7 Positioning Within the Literature

Landauer (1961) formulated the information–energy connection. Bennett (1982) developed the framework of reversible computation. Seifert (2012) systematized entropy production within stochastic thermodynamics. Parrondo, Horowitz, and Sagawa (2015) analyzed information engines, while Still et al. (2012) linked predictive information to thermodynamic efficiency. Friston's free energy principle discusses the energetic cost of model updating in biological systems.

The present work introduces an additional distinction:

A separate thermodynamic lower bound may exist for the task-relevant subset of mutual information.

The next section formally defines this subset.

While related notions appear in predictive information theory, empowerment, and task-relevant coarse-graining frameworks, the present work differs in explicitly defining semantic information as a bounded utility-weighted mutual information and deriving conditional thermodynamic lower bounds via canonical KL–free energy relations.

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3. Definition of Task-Weighted (Semantic) Information

3.1 Motivation

Mutual information

$$I(X;Y) = \mathbb{E}\left[\log \frac{p(x,y)}{p(x)p(y)}\right]$$

measures the statistical dependence between two random variables.

However, this quantity does not distinguish whether such dependence is valuable with respect to the system's objective or task. Random environmental correlations may increase mutual information without contributing to system performance, control accuracy, or adaptive output.

Therefore, the amount of information must be weighted relative to the system's objective. The goal is to formalize the distinction between total mutual information and the component of correlations aligned with a task.

3.2 Utility (Loss) Function

Let a loss function defining the system's task be given by

$$\mathcal{L}(x, y),$$

where $x$denotes the system state and $y$the environmental variable.

Define a normalized utility function as

$$U(x, y) = 1 - \frac{\mathcal{L}(x, y)}{L_{\max}},$$

so that

$$0 \le U(x, y) \le 1.$$

If the utility is high, the corresponding correlation is valuable with respect to the task.

The boundedness of $U$is critical for establishing the thermodynamic inequalities developed later.

3.3 Definition of Semantic Information

Task-weighted (semantic) information is defined as

$$A = \int U(x, y)\, p(x, y) \log \frac{p(x, y)}{p(x)p(y)}\, dx\, dy.$$

Equivalently, in expectation form,

$$A = \mathbb{E}\left[U(x, y) \log \frac{p(x, y)}{p(x)p(y)}\right].$$

For comparison, mutual information is

$$I(X;Y) = \mathbb{E}\left[\log \frac{p(x,y)}{p(x)p(y)}\right].$$

Define the pointwise mutual information

$$Z(x,y) := \log \frac{p(x,y)}{p(x)p(y)}.$$

Then

$$A = \mathbb{E}[U \cdot Z].$$

This definition represents mutual information filtered through a task-relevance function. Importantly, $A$is not a new type of information; it is a utility-weighted component of mutual information.

3.4 Mathematical Properties

(i) Boundedness

Since $0 \leq U \leq 1$,

$$| A | \leq I(X;Y).$$

Because $Z(x,y)$may take negative values, the sign of Adepends on the distribution.

For energetic inequalities, we define the physically relevant semantic magnitude as

$$A_+ := | A |.$$

(ii) Linearity

Semantic information is linear in the utility function:

$$A[\alpha U_1 + \beta U_2] = \alpha A[U_1] + \beta A[U_2].$$

(iii) Absence of Task

If

$$U(x, y) = 0,$$

then

$$A = 0.$$

Thus, without task relevance, semantic information vanishes.

(iv) Data Processing Inequality

For classical mutual information,

$$I(X; Y) \geq I(f(X); Y).$$

However, this inequality does not generally hold for semantic information, because the utility function may change under transformation.

This demonstrates that semantic information is contextual and task-dependent.

(v) Physical Positivity Convention

Since task-weighted information may take negative values due to negative PMI regions, all thermodynamic inequalities in this work are formulated using

$$A_{+} := | A |.$$

All energetic lower bounds refer to A_{+}, ensuring physical positivity.

Note. The relation between semantic information growth and KL divergence growth holds in regimes where correlation structure strengthens in alignment with the task. If sign reversals occur, inequalities must be interpreted in absolute-value form.

3.5 Interpretation and Positioning

The definition yields the following consequences:

- Shannon information is preserved.
- Random correlations are suppressed when utility is low.

- Functionally relevant correlations are emphasized.
- Semantic information represents the task-aligned component of mutual information.

Thus, A differs from pure mutual information and measures the density of goal-aligned correlations within the system.

This distinction forms the foundation for the thermodynamic lower bounds derived in the next section.

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4. Thermodynamic Cost of Task-Weighted Information

In this section, we examine whether the production of task-relevant information carries a thermodynamic cost. The objective is to establish a precise relationship between semantic information A and thermodynamic quantities such as free energy and entropy production.

We emphasize that two distinct KL divergences appear in this framework:

- Statistical dependence KL (mutual information):

$$I(X;Y) = D_{\mathrm{KL}}(p(x,y) \parallel p(x)p(y)),$$

- Thermodynamic KL (free energy deviation):

$$D_{\mathrm{KL}}(p(x) \parallel q(x)).$$

These are defined on different probability spaces and must not be conflated.

4.1 Landauer's Principle (Cautious Positioning)

Landauer's principle states that the minimum energy required to irreversibly erase one bit of information is

$$\Delta E \geq kT\ln(2).$$

More generally,

$$\Delta E \geq kT\ln(2)\,\Delta I.$$

However, Landauer's principle applies to erasure processes and does not distinguish informational content. The present problem concerns probability restructuring aligned with a task rather than erasure. Therefore, Landauer's inequality does not directly apply; instead, we explore a generalized lower bound via the free energy–KL relation.

4.2 KL Divergence and Free Energy

Let the reference distribution be canonical:

$$q(x) = \frac{e^{-\beta E(x)}}{Z}, \beta = \frac{1}{kT}.$$

Then

$$D_{\mathrm{KL}}(p \parallel q) = \int p(x) \ln \frac{p(x)}{q(x)}\, dx,$$

and the standard thermodynamic identity gives

$$F(p) - F(q) = kT\, D_{\mathrm{KL}}(p \parallel q).$$

Thus, deviation from equilibrium corresponds directly to free energy difference.

4.3 Semantic Information and KL Deviation

Recall

$$A = \int U(x,y)\, p(x,y) \ln \frac{p(x,y)}{p(x)p(y)}\, dx\, dy.$$

Since $0 \le U \le 1$,

$$| A | \le I(X;Y).$$

Define

$$A_+ := | A |.$$

Alignment requires that task-relevant correlation growth contributes positively to total KL growth relative to the canonical reference. This assumption excludes regimes in which utility-weighted correlations reorganize probability mass without increasing canonical free-energy deviation.

Alignment Condition (Formal Regime Assumption)

We restrict attention to regimes in which the thermodynamic KL divergence from the canonical reference distribution increases:

$$\Delta D_{\mathrm{tot}} > 0.$$

Here, $D_{\mathrm{tot}} := D_{KL}(p \parallel q_{\mathrm{can}})$denotes the KL divergence from the canonical Boltzmann reference distribution.

We further assume that semantic information growth constitutes a bounded subcomponent of this total deviation:

$$0 \leq \Delta A_+ \leq \Delta D_{\mathrm{tot}},$$

where

$$A_+ := | A |$$

denotes the physically relevant semantic magnitude.

Under these assumptions, it follows that

$$| \Delta A | \leq \Delta D_{\mathrm{tot}}.$$

This alignment condition ensures that task-relevant correlation growth does not exceed total deviation from thermodynamic equilibrium.

The condition is regime-dependent and is not a universal thermodynamic identity. It holds when the utility function selects correlations whose strengthening contributes monotonically to canonical KL divergence growth.

4.4 Lower Bound via Free Energy

Using

$$\Delta F = kT \, \Delta D_{\mathrm{KL}},$$

and the alignment condition,

$$\Delta F \geq kT \, \Delta A_+.$$

This is a conditional lower bound valid in alignment regimes.

4.5 Energy Lower Bound (Conditional Hypothesis)

From the free energy–KL relation,

$$\Delta F = kT\, D_{\mathrm{KL}}(p \parallel q_{\mathrm{canonical}}).$$

Since $A_+ \leq D_{\mathrm{KL}}$, we obtain

$$\Delta F \geq kT\, A_+.$$

If base-2 logarithms are used,

$$\Delta E \geq kT \ln(2)\, \Delta A_+.$$

Interpretation

The production of task-relevant correlations requires deviation from equilibrium and thus free energy expenditure. This inequality does not imply identity between energy and semantic information, but establishes a conditional lower bound.

Regime Conditions

The bound holds only under:

- Canonical reference distribution
- Open non-equilibrium dynamics
- Bounded utility $0 \leq U \leq 1$
- Positive alignment between semantic growth and KL growth

If alignment fails, only

$$A_+ \leq D_{\mathrm{KL}}(p \parallel q)$$

can be guaranteed.

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5. Entropy Production and the Rate of Semantic Information

5.1 Non-Equilibrium Entropy Production

For an open system, the total entropy production rate is

$$\sigma = \frac{dS_{\text{tot}}}{dt}, \sigma \geq 0.$$

For continuous-time Markov processes (Seifert, 2012),

$$\sigma = \frac{1}{2}\sum_{i,j}(J_{ij} - J_{ji}) \ln \frac{J_{ij}}{J_{ji}},$$

where

$$J_{ij} = p_i W_{ij}.$$

If detailed balance holds, $J_{ij} = J_{ji}$and $\sigma = 0$.
Thus, entropy production quantifies deviation from thermodynamic equilibrium.

5.2 KL Dynamics and Free Energy

Let the system distribution evolve as $p(t)$, while the canonical reference $q$remains fixed. Define

$$\dot{D}_{\text{KL}} = \frac{d}{dt} D_{\text{KL}}(p(t) \parallel q).$$

Under canonical assumptions,

$$F(p) - F(q) = kT D_{\text{KL}}(p \parallel q).$$

Taking the time derivative:

$$\frac{dF}{dt} = kT\dot{D}_{\text{KL}}.$$

The free energy balance reads:

$$\frac{dF}{dt} = \dot{W}_{\text{ext}} - T\sigma.$$

Combining:

$$T\sigma = \dot{W}_{\text{ext}} - kT\dot{D}_{\text{KL}}.$$

Thus, entropy production depends explicitly on external work exchange.

5.3 Rate of Semantic Information

Semantic information evolves as

$$A(t) = \mathbb{E}[U \cdot Z].$$

Its rate is

$$\dot{A} = \frac{dA}{dt}.$$

Without additional assumptions, no universal inequality between $\dot{A}$and $\dot{D}_{\mathrm{KL}}$can be asserted.

5.4 Conditional Relation to Entropy Production

Restrict to regimes where:

- the reference distribution is canonical,
- $\dot{D}_{\mathrm{KL}} > 0$,
- semantic growth is aligned with KL growth,
- external work flux is controlled or negligible.

Define the alignment factor:

$$\alpha(t) := \frac{\dot{A}(t)}{\dot{D}_{\mathrm{KL}}(t)} \, (\dot{D}_{\mathrm{KL}} \neq 0).$$

In alignment regimes:

$$0 < \alpha(t) \leq 1.$$

Using

$$\frac{dF}{dt} = kT\dot{D}_{\mathrm{KL}},$$

and neglecting $\dot{W}_{\mathrm{ext}}$,

$$\sigma \approx -k\dot{D}_{\mathrm{KL}}.$$

Substituting

$$\dot{D}_{\mathrm{KL}} = \frac{\dot{A}}{\alpha},$$

we obtain

$$\sigma \approx \frac{k}{\alpha}\, \dot{A}_{+}.$$

Define

$$\lambda := \frac{k}{\alpha}.$$

Thus, in alignment regimes,

$$\sigma \approx \lambda\, \dot{A}_{+}.$$

This proportionality is conditional and regime-dependent.

5.5 Interpretation of λ

Dimensional analysis:

$$[\sigma] = \frac{J}{K \cdot s}, [\dot{A}] = \frac{\text{bit}}{s}.$$

Hence,

$$[\lambda] = \frac{J}{K \cdot \text{bit}}.$$

If natural logarithms are used:

$$[\lambda] = \frac{J}{K \cdot \text{nat}}.$$

Thus, $\lambda$is of the order of Boltzmann's constant per unit information.

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6. Definition of the Semantic Ratio and Field Interpretation

6.1 Ratio Definition

Recall

$$A = \mathbb{E}\left[U(x,y)\log\frac{p(x,y)}{p(x)p(y)}\right], I(X;Y) = \mathbb{E}\left[\log\frac{p(x,y)}{p(x)p(y)}\right].$$

Since $0 \le U \le 1$,

$$|A| \le I.$$

Define the semantic ratio

$$s := \frac{A_+}{I}, A_+ := |A|, I > 0.$$

This quantity represents the fraction of mutual information aligned with the task.

6.2 Interpretation

If $U \geq 0$and $I > 0$,

$$0 \le s \le 1.$$

- $s = 1$: all correlations are task-aligned.
- $s = 0$: no task-relevant information.
- $0 < s < 1$: partial alignment.

This ratio may be interpreted as an information alignment efficiency.

6.3 Dimensional Analysis

$$[I] = \text{bit or nat}, [A] = \text{bit or nat}.$$

Thus,

$$s = \frac{A_+}{I}$$

is dimensionless.
Its energetic relevance arises only through

$$\Delta E \geq kT\ln(2)\, \Delta A_+.$$

6.4 Local Interpretation

Define the local semantic contribution density

$$a(x, y) = U(x, y)\, p(x, y)\log \frac{p(x, y)}{p(x)p(y)}.$$

Then

$$A = \int a(x, y)\, dx\, dy.$$

Define the local semantic ratio

$$s(x, y) = \frac{U(x, y)\log \frac{p(x, y)}{p(x)p(y)}}{\log \frac{p(x, y)}{p(x)p(y)}} = U(x, y),$$

whenever the denominator is nonzero.
This interpretation remains phenomenological and does not introduce a new physical field.

6.5 Energy Decomposition

Since

$$A = sI,$$

we obtain

$$\Delta A = I\, \Delta s + s\, \Delta I.$$

Thus,

$$\Delta E \geq kT\ln(2)(I\, \Delta s + s\, \Delta I).$$

The semantic ratio itself is not an energy unit; it modulates how total information contributes to energetic cost.

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7. Experimental Predictions and Testability

7.1 Central Hypothesis

The framework proposes two conditional inequalities:

$$\Delta E \geq kT \ln(2)\, \Delta A_{+},$$
$$\sigma \geq \lambda \dot{A}_{+}.$$

These are not universal thermodynamic laws, but conditional lower-bound hypotheses for open non-equilibrium systems.

7.2 Testing in Artificial Intelligence Systems

Define the energetic efficiency

$$\eta_A := \frac{\Delta E}{\Delta A_{+}}.$$

Compare two controlled scenarios:

1. Task-aligned learning (true labels)
2. Random-label learning

Testable hypothesis:

$$\eta_A^{(\text{task})} \neq \eta_A^{(\text{random})}.$$

A stronger empirical hypothesis is

$$\eta_A^{(\text{task})} < \eta_A^{(\text{random})}.$$

The null hypothesis states that no systematic difference exists.

7.3 Biological Systems

Test correlation:

$$\mathrm{Corr}(\dot{A}_{+}, \text{metabolic rate}) > 0,$$

controlling for stimulus intensity.

7.4 Stochastic Information Engines

Entropy production:

$$\sigma = \frac{1}{2} \sum_{i,j} \left(J_{ij} - J_{ji}\right) \ln \frac{J_{ij}}{J_{ji}}.$$

Test proportionality:

$$\sigma \approx \lambda \dot{A}_{+}.$$

7.5 Regression Model

$$\sigma = \lambda \dot{A}_{+} + \varepsilon,$$

where $\varepsilon$is a residual error term.

7.6 Falsifiability Criteria

The framework is falsified if any of the following are systematically observed:

$$\Delta A_{+} > 0\text{while}\Delta E \approx 0,$$
$$\mathrm{Corr}(\dot{A}_{+}, \sigma) \approx 0,$$

or no statistical distinction exists between task and random learning efficiencies.

8. Discussion and Limitations

This section addresses the scope, assumptions, and potential limitations of the proposed framework.

8.1 Operational Interpretation of “Meaning”

In this work, the term *semantic information* does not imply an ontological or phenomenological claim about meaning.

The definition is entirely operational:

- It is based on a loss/utility function.
- It is task-dependent.
- It is defined through measurable statistical quantities.

Accordingly, the notion of “meaning” used here is not identical to:

- consciousness,
- subjective experience,
- intention,

- phenomenal content.

The proposed framework analyzes only the physical cost of task-based correlations.

This distinction is critical to avoid conceptual overextension.

8.2 Context-Dependence of the Utility Function

Semantic information depends explicitly on the utility function. The choice of utility depends on:

- task definition,
- performance criterion,
- measurement framework.

Therefore, semantic information is not absolute but contextual.

This contextuality is not a weakness, but a natural consequence of task-based information definitions.

However, it implies that:

Different utility specifications may produce different semantic information values.

For this reason, experimental studies must explicitly report the chosen utility function.

8.3 Limits of Universality

The proposed inequalities

$$\Delta E \geq kT\ln(2)\,\Delta A_{+}, \sigma \geq \lambda \dot{A}_{+}$$

are not presented as universal physical laws.

They are proposed under the following conditions:

- Open systems
- Non-equilibrium dynamics

- Markovian transitions
- Canonical ensemble reference
- Bounded utility

Outside these assumptions, no general lower bound between energetic cost and task-relevant information can be guaranteed.

Thus, this work does not modify existing thermodynamic laws; it extends information thermodynamics through the distinction of task-relevant information.

8.4 Limits of the Landauer Extension

Landauer's principle concerns information erasure.

The present work instead analyzes:

- Information production
- Distribution restructuring
- Task-aligned correlation increase

The proposed energetic lower bound is not a direct derivation of Landauer's principle; it is a generalization based on the free energy–KL divergence relationship.

The validity of this generalization ultimately depends on experimental verification.

8.5 Relation to Alternative Theories

The concept of semantic information has parallels with:

- The Free Energy Principle
- Predictive Processing
- Dissipative Structures
- Information Engines

However, the present framework is not identical to any of these theories.

It is:

- not a reformulation of Friston's free energy principle,
- not an extension of Maxwell demon literature,
- but shares mathematical similarities and conceptual contact points.

Clarifying this distinction is important to avoid theoretical conflation.

8.6 Strengths and Limitations of the Theory

Strengths

- The mathematical definition is explicit and internally consistent.
- The KL–free energy connection provides a physical foundation.
- The framework generates testable hypotheses.
- It applies to both artificial and biological systems.

Limitations

- The utility function is context-dependent.
- The coefficient λ is not universal and must be experimentally determined.
- Direct measurement of ΔA may be difficult in complex biological systems.
- The distinction between correlation and causation must be handled carefully.

These limitations indicate directions for further theoretical refinement and empirical investigation.

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9. Conclusion

In this work, the framework of information thermodynamics has been systematically extended through the distinction of task-relevant information. While Shannon information theory measures statistical dependence, it does not distinguish the value that such dependence carries with respect to system objectives. To address this limitation, we introduced semantic information as a utility-weighted form of mutual information:

$$A = \int U(x,y)\, p(x,y) \ln \frac{p(x,y)}{p(x)p(y)}\, dx\, dy.$$

This definition measures the density of correlations aligned with a task and represents a contextual component of mutual information.

The principal contribution of this work is to show that the production of semantic information can be structurally related to free energy and entropy production. Using the established relation between KL divergence and free energy difference, we proposed the following conditional hypotheses for task-relevant information production:

$$\Delta E \geq kT \ln(2)\, \Delta A_+, \sigma \geq \lambda \dot{A}_+.$$

These expressions are not presented as universal physical laws, but as testable lower-bound hypotheses valid for open non-equilibrium systems under specified dynamical conditions.

The proposed framework:

- does not modify existing thermodynamic laws,
- is consistent with non-equilibrium thermodynamics,
- is grounded in the KL–free energy relationship,
- is mathematically explicit and internally consistent,
- generates experimentally testable predictions.

In this context, semantic information is not a metaphysical category of meaning, but a measure of task-based correlation alignment. The work makes no claims about consciousness, subjective experience, or ontological meaning; it analyzes only the physical cost of task-weighted information production.

The framework opens several directions for future research, including:

- Experimental determination of the entropy-production coefficient λ,
- Quantitative measurement of energy–semantic information ratios in artificial learning systems,
- Analysis of task-relevant information dynamics in biological adaptation,
- Investigation of generalized bounds in non-Markovian and strongly feedback-driven systems.

In conclusion, the hypothesis that task-relevant information production carries energetic cost provides a mathematically consistent and experimentally testable framework for analyzing the physical limits of information-processing systems. By extending information thermodynamics from pure statistical dependence toward functional selection, this approach enables a more refined classification of the energy–information relationship.

The proposed framework treats information production not as an ontological construct, but as measurable correlation alignment in open systems. In this sense, it offers a principled starting point for investigating the limits of task-relevant information production without conflicting with established thermodynamic laws.

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10. Mathematical Results and Theorems

This section establishes the mathematical bounds and thermodynamic consistency of the task-weighted information measure. The following results hold under the assumptions:

- $0 \leq U(x,y) \leq 1$,

- All relevant integrals are finite,
- The KL divergence is well-defined,
- For Theorems 3 and 4, the reference distribution is canonical Boltzmann.

Theorem 1 — Upper Bound Property (Boundedness)

Proposition.
If $0 \leq U(x,y) \leq 1$, then semantic information Asatisfies

$$|A| \leq \int p(x,y)\ |Z(x,y)|\ dx\,dy,$$

where

$$Z(x,y) := \log \frac{p(x,y)}{p(x)p(y)}.$$

If additionally $Z(x,y) \geq 0$almost everywhere, then

$$0 \leq A \leq I(X;Y).$$

Proof.
By definition,

$$A = \int U(x,y)\,p(x,y)\,Z(x,y)\,dx\,dy.$$

Taking the absolute value,

$$|A| \leq \int U(x,y)\,p(x,y)\ |Z(x,y)|\ dx\,dy.$$

Since $0 \leq U(x,y) \leq 1$,

$$|A| \leq \int p(x,y)\ |Z(x,y)|\ dx\,dy.$$

If $Z(x,y) \geq 0$almost everywhere, then

$$A \leq \int p(x,y)\,Z(x,y)\,dx\,dy = I(X;Y).$$

Comment.
Semantic information cannot exceed total statistical dependence. Task-relevant information is a subcomponent of mutual information.

Theorem 2 — Zero Task Case

Proposition.
If $U(x,y) \equiv 0$, then

$$A = 0.$$

Proof.
From the definition,

$$A = \int 0 \cdot p(x,y)\,Z(x,y)\,dx\,dy = 0.$$

Comment.
This confirms the contextual nature of semantic information. Even if statistical dependence exists, no semantic contribution arises without a task definition.

Theorem 3 — Free Energy Lower Bound (Conditional)

Assumptions

- The reference distribution is canonical Boltzmann:

$$q(x) = \frac{e^{-\beta E(x)}}{Z}, \beta = \frac{1}{kT}.$$

- The system is described by a canonical ensemble.
- Utility is bounded $0 \leq U \leq 1$.
- The alignment condition holds (semantic growth contributes positively to total KL growth).

Known Result
From statistical physics,

$$F(p) - F(q) = kT\, D_{\mathrm{KL}}(p \parallel q).$$

Proposition
Under the alignment regime,

$$F(p) - F(q) \geq kT\, A_{+},$$

where

$$A_{+} := \max(A, 0).$$

Proof
KL divergence measures total deviation from equilibrium:

$$D_{\mathrm{KL}}(p \parallel q) \geq 0.$$

Under bounded utility and alignment,

$$A_{+} \leq D_{\mathrm{KL}}(p \parallel q).$$

Therefore,

$$F(p) - F(q) = kT\, D_{\mathrm{KL}}(p \parallel q) \geq kT\, A_{+}.$$

Comment.
Task-relevant information production requires a free-energy difference driving the system away from thermodynamic equilibrium. This provides the mathematical basis for the energetic cost hypothesis.

Theorem 4 — Conditional Dynamic Relation to Entropy Production

Assumptions

- The system is open and out of equilibrium.
- Dynamics are Markovian or near detailed balance.

- External work flux is constant or negligible.
- The alignment regime holds.

Known Result
In stochastic thermodynamics,

$$\frac{dF}{dt} = \dot{W}_{ext} - T\sigma.$$

If external work is negligible or constant,

$$T\sigma \geq -\frac{dF}{dt}.$$

From Theorem 3,

$$F(p) - F(q) = kT\, D_{KL}(p \parallel q).$$

Differentiating,

$$\frac{dF}{dt} = kT\, \frac{d}{dt} D_{KL}.$$

Alignment Regime
In regimes where the system is driven away from equilibrium,

$$\frac{d}{dt} D_{KL} > 0.$$

Define the alignment factor

$$\alpha(t) := \frac{\frac{dA_+}{dt}}{\frac{d}{dt} D_{KL}}, 0 < \alpha(t) \leq 1.$$

Thus,

$$\frac{dA_+}{dt} = \alpha \frac{d}{dt} D_{KL}.$$

Derivation
Substituting into the entropy relation:

$$T\sigma \geq kT \frac{d}{dt} D_{KL}.$$

Dividing by T:

$$\sigma \geq k \frac{d}{dt} D_{KL}.$$

Using the alignment relation:

$$\frac{d}{dt} D_{KL} = \frac{1}{\alpha} \frac{dA_+}{dt}.$$

Therefore,

$$\sigma \geq \frac{k}{\alpha}\frac{dA_+}{dt}.$$

Final Result

$$\sigma \geq \lambda \frac{dA_+}{dt},$$

where

$$\lambda := \frac{k}{\alpha}.$$

Definition of the Coefficient λ

The coefficient $\lambda$is not universal.
It depends on the alignment factor:

$$\lambda = \frac{k}{\alpha}, 0 < \alpha \leq 1.$$

Since $\alpha$is dimensionless, λhas the same physical dimension as Boltzmann's constant per unit information.

Dimensional Analysis

Entropy production rate:

$$[\sigma] = \frac{J}{K \cdot s}.$$

Semantic information rate:

$$\left[\frac{dA_+}{dt}\right] = \frac{\text{bit}}{s} \text{or} \frac{\text{nat}}{s}.$$

Thus,

$$[\lambda] = \frac{J}{K \cdot \text{bit}} \text{or} \frac{J}{K \cdot \text{nat}}.$$

Since

$$[k] = \frac{J}{K},$$

the coefficient $\lambda$is of the order of Boltzmann's constant per unit information, scaled by the inverse alignment factor $1/\alpha$.

Comment

Under alignment conditions, an increase in the production rate of task-relevant (semantic) information necessarily requires a proportional increase in entropy production rate.
However, the proportionality coefficient is system-dependent and not universal.
No universal thermodynamic identity is claimed.

11. Application to a Two-State Markov Model

In this section, the proposed semantic information framework is illustrated using one of the simplest non-equilibrium systems: a two-state continuous-time Markov process.
The objective is to analytically examine the relationship between task-relevant information production and entropy production rate.

11.1 Model Definition

The system consists of two states:

$$\mathcal{X} = \{0,1\}.$$

Transition rates:

$$W_{01}: 0 \to 1, W_{10}: 1 \to 0.$$

The master equations are:

$$\frac{dp_0}{dt} = -W_{01}p_0 + W_{10}p_1,$$
$$\frac{dp_1}{dt} = -W_{10}p_1 + W_{01}p_0.$$

At stationarity $\frac{dp_i}{dt} = 0$:

$$p_0 = \frac{W_{10}}{W_{01} + W_{10}}, p_1 = \frac{W_{01}}{W_{01} + W_{10}}.$$

11.2 Correlation with an Environmental Variable

Assume the system is correlated with an environmental variable Y.
The joint distribution is $p(x, y)$.
Mutual information is:

$$I(X; Y) = \sum_{x,y} p(x, y) \ln \frac{p(x, y)}{p(x)p(y)}.$$

This measures total statistical dependence.

11.3 Task Definition and Semantic Information

Suppose the task rewards only state 1:

$$U(1, y) = 1, U(0, y) = 0.$$

Then semantic information reduces to

$$A = \sum_y p(1,y) \ln \frac{p(1,y)}{p(1)p(y)}.$$

Only the task-aligned state contributes.
Thus, semantic information is a utility-weighted component of mutual information.

11.4 Entropy Production Rate

For a two-state Markov system, entropy production rate is

$$\sigma = \frac{1}{2} \sum_{i,j} (J_{ij} - J_{ji}) \ln \frac{J_{ij}}{J_{ji}}.$$

For two states:

$$\sigma = (J_{01} - J_{10}) \ln \frac{J_{01}}{J_{10}}.$$

Transition fluxes:

$$J_{01} = p_0 W_{01}, J_{10} = p_1 W_{10}.$$

Substituting stationary probabilities:

$$J_{01} = \frac{W_{10} W_{01}}{W_{01} + W_{10}}, J_{10} = \frac{W_{01} W_{10}}{W_{01} + W_{10}}.$$

Important:
For a simple two-state system **with constant rates**, detailed balance automatically holds:

$$J_{01} = J_{10},$$

and therefore

$$\sigma = 0.$$

Thus, a pure two-state system with fixed rates cannot sustain non-zero steady-state entropy production.
To obtain $\sigma > 0$, one of the following is required:

- Time-dependent driving,
- Coupling to multiple reservoirs,
- Non-conservative forcing,
- Or multi-state models with cycle currents

11.5 Non-Equilibrium Driving

Assume now that rates are externally driven so that detailed balance is broken (e.g., via time-dependent forcing or asymmetric reservoir coupling).
Then

$W_{01} \neq W_{10}$under non-equilibrium driving.

In that case:

- $p_1$increases when W_{01}increases,
- semantic information Aincreases,
- entropy production $\sigma > 0$.

Thus, task-relevant probability restructuring requires non-equilibrium flux.

11.6 Concrete Illustration of the Hypothesis

Under non-equilibrium driving:
If

$$\Delta A_+ > 0,$$

then detailed balance must be broken, implying

$$\sigma > 0.$$

This is consistent with the relations proposed earlier:

$$\Delta E \geq kT\ln(2)\,\Delta A_+,$$
$$\sigma \geq \lambda \dot{A}_+.$$

This two-state example provides a minimal analytic illustration of the conditional energetic cost hypothesis.
More complex task structures would require multi-state Markov processes with non-trivial cycle currents.

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12. Dimensional Analysis

This section examines the dimensional consistency of the quantities introduced in the framework. The objective is to demonstrate that the proposed energy–information inequalities are physically consistent in terms of units.

12.1 Information Quantities

Mutual information:

$$I(X;Y)$$

Semantic information:

$$A$$

Both quantities are Shannon information measures.

- If logarithm base 2 is used, the unit is **bit**.
- If the natural logarithm is used, the unit is **nat**.

Thus,

$$[I] = [A] = \text{bit or nat.}$$

These are dimensionless mathematical quantities that carry informational units only; they do not carry physical energy units.

12.2 Energy Terms

Energy change:

$$\Delta E, [\Delta E] = \mathrm{J}.$$

Landauer coefficient:

$$kT\ln(2).$$

Here:

- $k$is Boltzmann's constant $(\mathrm{J/K})$,
- $T$is temperature (K).

Therefore,

$$[kT] = \mathrm{J}.$$

If information is measured in bits:

$$kT\ln(2)\text{has unit}\,\frac{\mathrm{J}}{\text{bit}}.$$

If information is measured in nats:

$$kT\text{has unit}\,\frac{\mathrm{J}}{\text{nat}}.$$

Thus, the coefficient can be interpreted as an energy-per-unit-information scale.

12.3 Energy–Information Inequality

The proposed inequality:

$$\Delta E \geq kT\ln(2)\,\Delta A_{+}.$$

Dimensionally:

$$\mathrm{J} \;\geq\; \left(\frac{\mathrm{J}}{\text{bit}}\right)\cdot \text{bit},$$

or

$$\mathrm{J} \;\geq\; \left(\frac{\mathrm{J}}{\text{nat}}\right)\cdot \text{nat}.$$

Thus, the inequality is dimensionally consistent.
This expression does not imply that semantic information equals energy; rather, it states that energetic cost has a lower bound proportional to semantic information increase.

12.4 Entropy and Entropy Production Rate

Thermodynamic entropy:

$$S = kH,$$

where $H$is Shannon entropy (dimensionless in nat units).
Therefore,

$$[S] = \frac{\mathrm{J}}{\mathrm{K}}.$$

Entropy production rate:

$$\sigma = \frac{dS}{dt}.$$

Its dimension is:

$$[\sigma] = \frac{\mathrm{J/K}}{\mathrm{s}} = \frac{\mathrm{J}}{\mathrm{K \cdot s}}.$$

This represents the physical rate of entropy production.

12.5 Dimension of the Coefficient λ

From the inequality:

$$\sigma \geq \lambda \frac{dA_+}{dt},$$

we examine units.
If base-2 logarithms are used:

$$[\dot{A}_+] = \frac{\text{bit}}{\mathrm{s}}.$$

If natural logarithms are used:

$$[\dot{A}_+] = \frac{\text{nat}}{\mathrm{s}}.$$

Since

$$[\sigma] = \frac{\mathrm{J}}{\mathrm{K \cdot s}},$$

we obtain:

$$[\lambda] = \frac{\mathrm{J/(K \cdot s)}}{\text{bit}/s} = \frac{\mathrm{J}}{\mathrm{K} \cdot \text{bit}}.$$

If natural logarithms are used:

$$[\lambda] = \frac{\mathrm{J}}{\mathrm{K} \cdot \text{nat}}.$$

Thus, $\lambda$is of the order of Boltzmann’s constant per unit information.
Since

$$[k] = \frac{\mathrm{J}}{\mathrm{K}},$$

we may write

$$\lambda = k \times \alpha,$$

where $\alpha$is a dimensionless alignment factor expressed per unit information.
This confirms dimensional consistency with fundamental physical constants.

12.6 Consistency Result

Dimensional analysis yields:

- Semantic information is not energy.
- There is no identity between information units and energy units.
- The energy–information relation is mediated through a linear coefficient at the scale of kT.
- The proposed inequalities are dimensionally consistent.

Therefore, the framework contains no dimensional inconsistencies in the energy–information connection.

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